

Ionization of Atomic Hydrogen in Strong Infrared Laser Fields

Presented by: Brant Abeln ¹

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May 27, 2010

DAMOP 2010

Where is Drake University?



Outline

- 1 Introduction
 - Ultrafast Physics
 - Laser Pulse
- 2 Analysis
 - Observables
 - Numerical Methods
 - Matrix Iteration
- 3 New Results
 - Comparisons
 - New Calculations
- 4 Conclusions and Outlook

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Attosecond/Femtosecond Physics

Definition

1 Attosecond is one-millionth of one millionth of one millionth (10^{-18}) of a second.

- There are twice as many attoseconds in 1 second than seconds in the **age of the universe** (15 billion years)!
- The period for the $n = 1$ orbit in atomic hydrogen is ≈ 150 attoseconds
- Attosecond laser pulses provide a window to study the details of (valence) electron interactions in atoms and molecules.
- A major role for **theory** in attosecond science is to **explain novel ways to investigate and to control electronic processes** in matter on such ultra-short time scales.

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Hydrogen Atom in Strong Infrared Laser Pulse

- Intensity range from $10^{12} - 10^{15} \text{ W/cm}^2$
- 10^{14} W/cm^2 is a million billion times stronger than the radiation that the Earth gets from the Sun directly above us on a clear day.
- Such intensities can rip electrons away from atoms in a very different way from the standard photoeffect:
 - Multi-photon ionization
 - Above-threshold ionization
 - Field (tunnel) ionization
- Keldysh Parameter (for atomic hydrogen):

$$\gamma \approx \frac{850}{\lambda[\text{nm}] \sqrt{I[10^{14} \text{ W/cm}^2]}}$$

$\gamma \gg 1 \rightarrow$ multi-photon ionization; $\gamma < 0.5 \rightarrow$ tunnel ionization

$\gamma \approx 1 \rightarrow$ no clear picture and treatment becomes very difficult

$\gamma = 1.06$ for $\lambda = 800 \text{ nm}$ and $I = 10^{14} \text{ W/cm}^2$!!!

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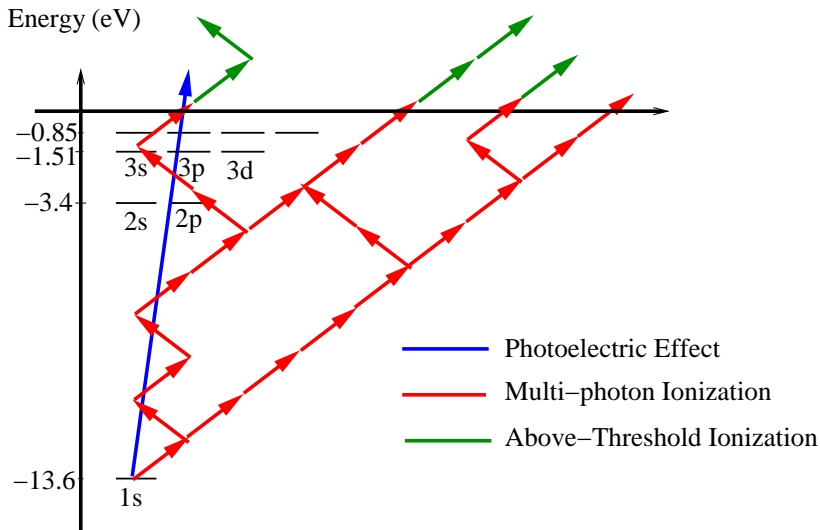
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Single vs. Multi-Photon Ionization in Atomic Hydrogen



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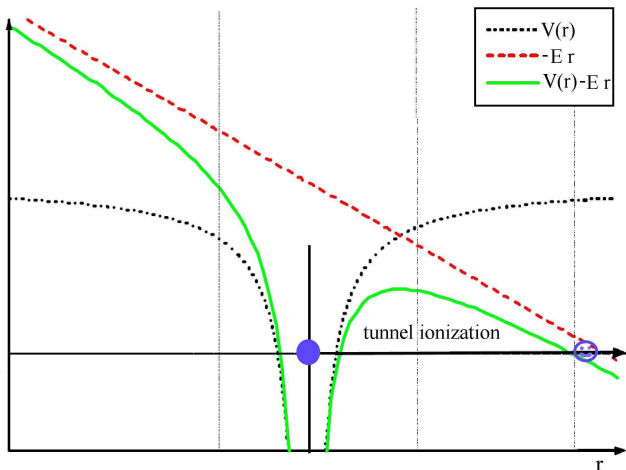
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Field (Tunnel) Ionization



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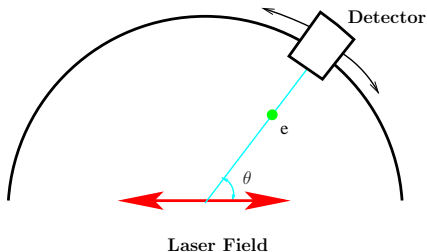
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Observables

- Energy spectrum
- Angular distribution
- Time-resolved visualization

Scheme of an Angular-Distribution Experiment



Gauge

Time-Dependent Schrödinger Equation

$$\hat{H}\Psi = i\frac{\partial}{\partial t}\Psi \quad (1)$$

Length form of electric dipole operator

$$\hat{H} = -\frac{1}{2}\nabla^2 + \frac{\ell(\ell+1)}{2r^2} - \frac{1}{r} + r\cos(\vartheta)E(t) \quad (2)$$

Velocity form of electric dipole operator

$$\hat{H} = -\frac{1}{2}\nabla^2 + \frac{\ell(\ell+1)}{2r^2} - \frac{1}{r} - \frac{i\mathbf{A}(t)}{c} \cdot \nabla \quad (3)$$

Propagation

- Finite difference
 - Crank-Nicholson
 - Matrix Iteration
 - Leap-Frog
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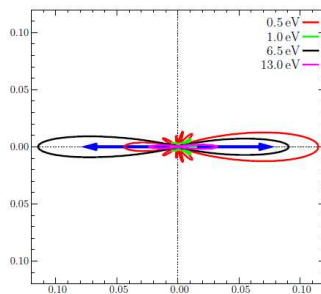
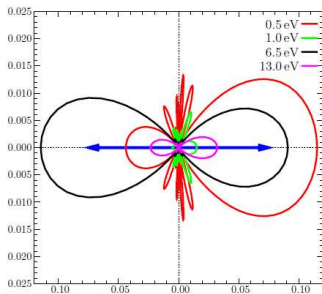
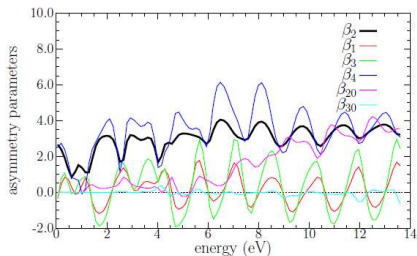
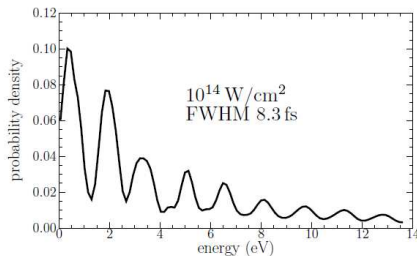
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Length Form



Matrix Iteration (MIM)

Crank-Nicholson Approximation

$$\Psi(\mathbf{r}, t + \Delta t) \approx \frac{1 - i\hat{H}\Delta t/2}{1 + i\hat{H}\Delta t/2} \Psi(\mathbf{r}, t) \quad (4)$$

- $1 + i\hat{H}\Delta t/2 = \hat{O}_D + \hat{O}_{ND}$
- $[1 + i\hat{H}\Delta t/2]^{-1} \approx (1 - \hat{O}_D^{-1}\hat{O}_{ND} + \hat{O}_D^{-1}\hat{O}_{ND}\hat{O}_D^{-1}\hat{O}_{ND} + \dots)\hat{O}_D^{-1}$
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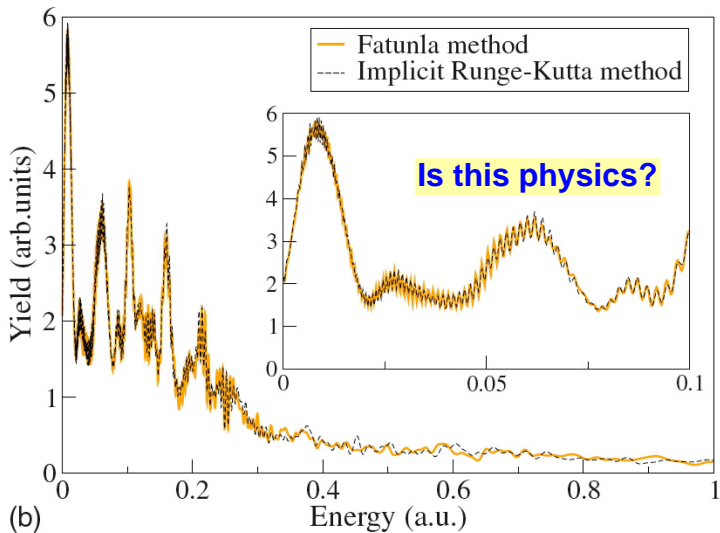
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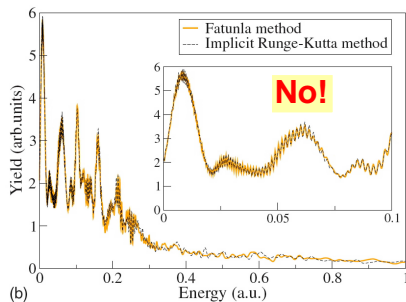
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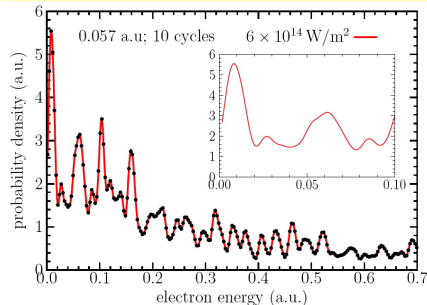
Madronero and Piraux



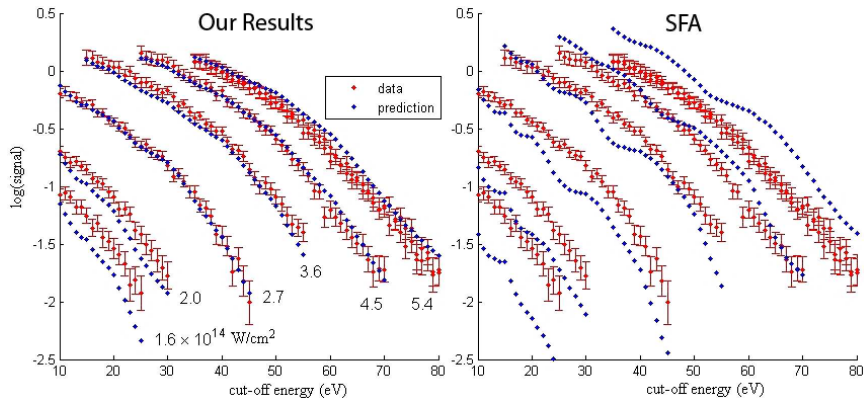
Comparisons



Our results show no numerical wiggles but more physics at high energies.



Comparison with Experiments



Conclusions

- IR laser fields calculations are **computationally expensive**.
- **MIM** yields numerically stable results.
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- Parallelize computer code (each run takes several days on single CPU)
- Look at more complex systems (Li, maybe H_2)

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More?

PHYSICAL REVIEW A **81**, 043408 (2010)

Ionization of atomic hydrogen in strong infrared laser fields

Alexei N. Grum-Grzhimailo,^{*} Brant Abeln,[†] Klaus Bartschat,[‡] and Daniel Weflen[§]

Department of Physics and Astronomy, Drake University, Des Moines, Iowa 50311, USA

Timothy Urness^{||}

Department of Mathematics and Computer Science, Drake University, Des Moines, Iowa 50311, USA

(Received 27 January 2010; published 14 April 2010)

We have used the matrix iteration method of Nurhuda and Faisal [Phys. Rev. A **60**, 3125 (1999)] to treat ionization of atomic hydrogen by a strong laser pulse. After testing our predictions against a variety of previous calculations, we present ejected-electron spectra as well as angular distributions for few-cycle infrared laser pulses with peak intensities of up to 10^{15} W/cm². It is shown that the convergence of the results with the number of partial waves is a serious issue, which can be managed in a satisfactory way by using the velocity form of the electric dipole operator in connection with an efficient time-propagation scheme.

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